**Exercise #7: The Discrete Fourier Transform (DFT)**

Submit 5 out of the 7 questions

1. **Resolution and sampling**
   1. What is the frequency resolution if you sample 3 seconds at 100 Hz?
   2. What is the frequency resolution if you sample 1000 points at 200 points/sec?
   3. How the resolution changes if you:
      1. Double the sampling rate
      2. Double the number of points you sample
2. Assume you computed a 8 points DFT in one second on signal x to obtain X. what you expect the X values to be if (verify with fft in matlab):
   1. x=2\*sin(2\*pi\*t\*2)
   2. x=3\*cos(2\*pi\*t\*3)
   3. x=2\*sin(2\*pi\*t\*2)+3\*cos(2\*pi\*t\*3)
   4. 5\*sin(2\*pi\*t\*5); (note the sampling rate)
   5. **Explain your answer.**
3. Let x=2\*sin(2\*pi\*t\*2)+3\*cos(2\*pi\*t\*3) (a matlab question)
   1. Sample it with 10 points in one second.
   2. Use the forward and inverse Fourier transform (as we did in class to remove the 50Hz noise) to remove the 2Hz signal
   3. Same but for removing the 3Hz signal
4. **Leakage**
   1. Let x=sin(2\*pi\*3\*n/N). Assume the sampling rate is 32 points/second and that you sample N=16.
   2. What is the duration of the signal you sampled?
   3. Plot the DFT (abs value) of x and explain why you don’t see the 3 Hz.
   4. How many more points are required in order to clearly view the 3Hz signal?
5. **The shift theorem**. A matlb question
   1. Assume x is a sinus signal of 2 Hz. Sample it with equally spaced 12 points (e.g. 0:1/12:1-1/12).
   2. Compute the X (the DFT of x). which values are not zero? Why?
   3. Shift the signal by 3 points in the time domain (e.g. x2=circshift(x',3) ) and plot this shifted signal (x2) on top of the original signal. From observing the signals you plotted, what is the phase shift the new signal underwent?
   4. Compute the shift factor of the 2 Hz frequency (e.g divide the DFT X2[3] of the shifted signal by the values X[3] of the DFT of the original signal). What is the phase shift value ?
   5. Compute the shift factor from the formula we learned in class: exp(-i\*2\*pi\*k\*m/N) and verify it is the same you obtained in d.
6. Let x🡨🡪X be a DFT pair. Describe how will X change if:
7. We add to x a constant ***a***
8. We multiply x by a real number ***b***
9. **Phase analysis** (this is a difficult question, 10 points bonus, use matlab)
   1. Consider the equation x=sin(2\*pi\*t) + sin(2\*pi\*t+pi/4). Find the values of ***a*** and ***p*** such that x = a\*sin(2\*pi\*t+p). (hint, check the power and phase of the DFT)